

Characteristic time scales of tick quotes on foreign currency markets: an empirical study and agent-based model

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Received 29 September 2005 / Received in final form 5 December 2005

Published online 12 April 2006 – © EDP Sciences, Società Italiana di Fisica, Springer-Verlag 2006

Abstract. Power spectrum densities for the number of tick quotes per minute (market activity) on three currency markets (USD/JPY, EUR/USD, and JPY/EUR) for periods from January 1999 to December 2000 are analyzed. We find some peaks on the power spectrum densities at a few minutes. We develop the double-threshold agent model and confirm that stochastic resonance occurs for the market activity of this model. We propose a hypothesis that the periodicities found on the power spectrum densities can be observed due to stochastic resonance.

PACS. 89.65.Gh Economics; econophysics, financial markets, business and management – 87.15.Ya Fluctuations – 02.50.-r Probability theory, stochastic processes, and statistics

1 Introduction

In the past few years there has been increasing interest in the investigation of financial markets as complex systems in statistical mechanics. The empirical analysis of the high frequency financial data reveals nonstationary statistics of market fluctuations and several mathematical models of markets based on the concept of the nonequilibrium phenomena have been proposed [1, 2].

Recently Mizuno et al. investigate high frequency data of the USD/JPY exchange market and conclude that dealers' perception and decision are mainly based on the latest 2 min data [3]. This result means that there are feedback loops of information in the foreign currency market.

As microscopic models of financial markets some agent models are proposed [4–7]. Specifically Ising-like models are familiar to statistical physicists and have been examined in the context of econophysics. The analogy to the paramagnetic-ferromagnetic transition is used to explain crashes and bubbles. Krawiecki et al. consider the effect of a weak external force acting on the agents in the Ising model of the financial market and conclude that apparently weak stimuli from outside can have potentially effect on financial market due to stochastic resonance [8]. This conclusion indicates that it is possible to observe the effect of the external stimuli from the market fluctuations.

Motivated by their conclusion we investigate high-frequency financial data and find a potential evidence that stochastic resonance occurs in financial markets. In this article the results of data analysis are reported and the

agent-based model is proposed in order to explain this phenomenon.

2 Analysis

We analyze tick quotes on three foreign currency markets (USD/JPY, EUR/USD, and JPY/EUR) for periods from January 1999 to December 2000 [9]. This database contains time stamps, prices, and identifiers of either ask or bid. Since generally market participants (dealers) must indicate both ask and bid prices in foreign currency markets the nearly same number of ask and bid offering are recorded in the database. Here we focus on the ask offering and regard the number of ask quotes per unit time (one minute) $A(t)$ as the market activity. The reason why we define the number of ask quotes as the market activity is because this quantity represents amount of dealers' responses to the market.

In order to elucidate temporal structure of the market activity power spectrum densities of $A(t)$, estimated by

$$S(f) = \frac{1}{2\pi} \lim_{T \rightarrow \infty} \frac{1}{T} \left\langle \left| \int_0^T A(\tau) e^{-2\pi i f \tau} d\tau \right|^2 \right\rangle, \quad (1)$$

where f represents frequency, and T a maximum period of the power spectrum density, are calculated. Figures 1–3 show the power spectrum densities for three foreign currency markets (USD/JPY, EUR/USD, and EUR/JPY) from January 1999 to December 2000. It is found that they have some peaks at the high frequency region.

There is a peak at 2.5 min on the USD/JPY market, at 3 min on the EUR/USD market, and there are

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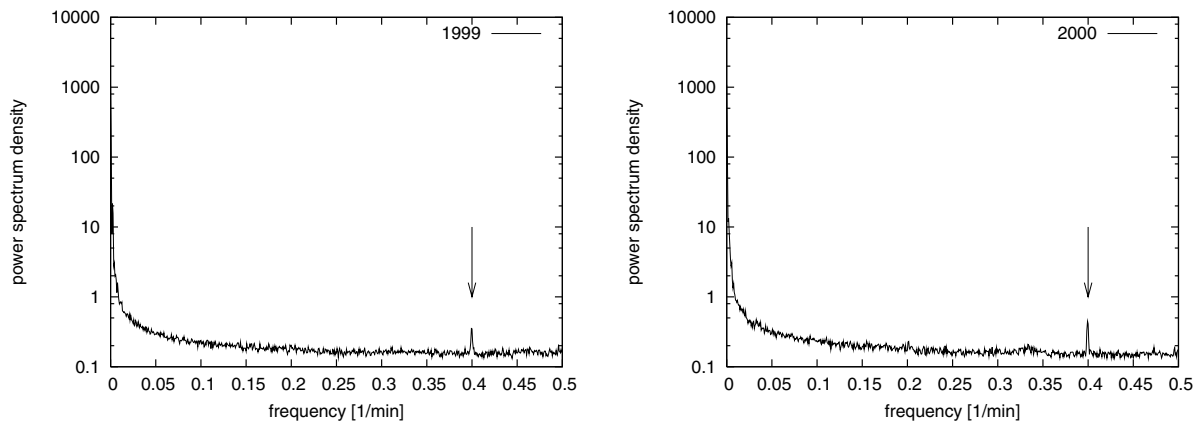


Fig. 1. Semi-log plots of power spectrum densities for time series of the number of ask quotes per minute on the USD/JPY market on 1999 (left) and 2000 (right). These power spectrum densities are estimated by averaging power spectrum densities for intraday time series of the number of ask quotes per minute over day for each year.

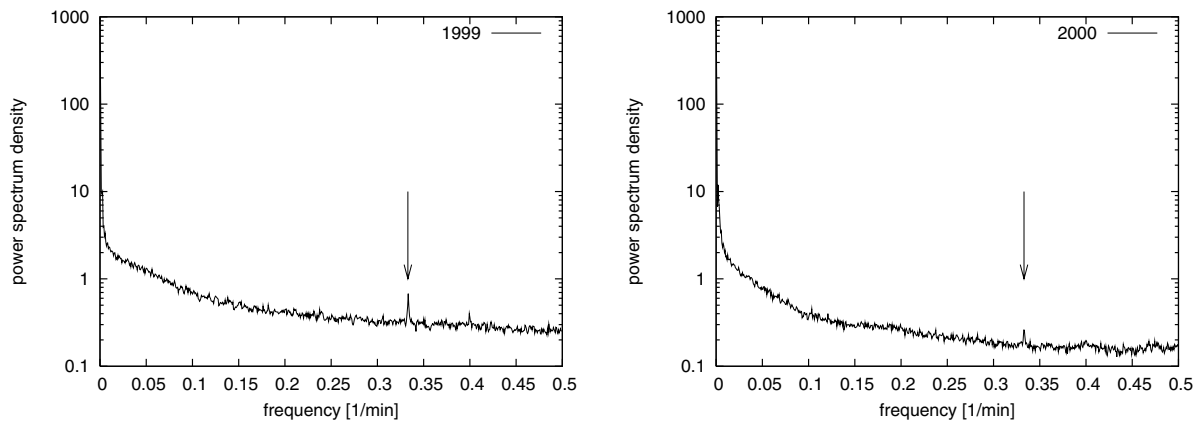


Fig. 2. Semi-log plots of power spectrum densities for time series of the number of ask quotes per minute on the EUR/USD market on 1999 (left) and 2000 (right).

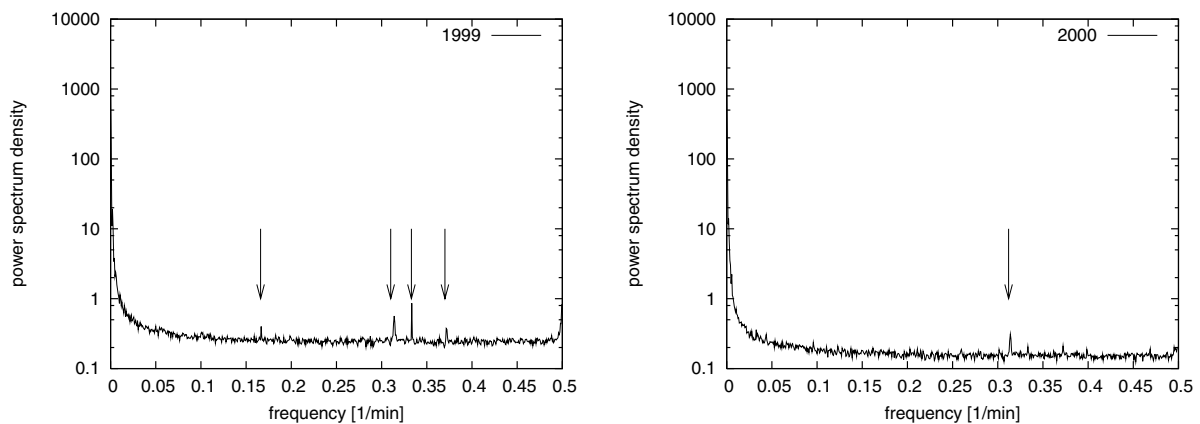


Fig. 3. Semi-log plots of power spectrum densities for time series of the number of ask quotes per minute on the EUR/JPY market on 1999 (left) and 2000 (right).

some peaks on the JPY/EUR. We confirm that these peaks appear and disappear depending on observation periods. On the USD/JPY market there is the peak for periods of January 1999–July 1999, March 2000–April 2000, and August 2000–November 2000; on the EUR/USD market July 1999–September 1999; and on the EUR/JPY market January 1999–March 1999, April 1999–June 1999, November 1999, and July 2000–December 2000.

These peaks mean that market participants offer quotes periodically and in synchronization. The possible reasons for these peaks to appear in the power spectrum densities of the market activity are follows:

1. The market participants are affected by common periodic information.
2. The market participants are spontaneously synchronized.

In the next section the double-threshold agent model is introduced and explain this phenomenon on the basis of the reason (1).

3 Double-threshold agent model

Here we consider a microscopic model for financial markets in order to explain the dependency of the peak height on observation periods. We develop the double-threshold agent model based on the threshold dynamics.

In foreign exchange markets the market participants attend the markets with utilizing electrical communication devices, for example, telephones, telegrams, and computer networks. They are driven by both exogenous and endogenous information and determine their investment attitudes. Since the information to motivate buying and one of selling are opposite to each other we assume that the information is a scalar variable. Moreover the market participants perceive the information and determine their investment attitude based on the information. The simplest model of the market participant is an element with threshold dynamics.

We consider a financial market consisting of N market participants having three kinds of investment attitudes: buying, selling, and doing nothing. Recently we developed an array of double-threshold noisy devices with a global feedback [10]. Applying this model to the financial market we construct three decision model with double thresholds and investigate the dependency of market behavior on an exogenous stimuli.

The investment attitude of the i th dealer $y_i(t)$ at time t is determined by his/her recognition for the circumstances $x_i(t) = s(t) + z_i(t)$, where $s(t)$ represents the investment environment, and $z_i(t)$ the i th dealer's prediction from historical market data. $y_i(t)$ is given by

$$y_i(t) = \begin{cases} 1 & (x_i(t) + \xi_i(t) \geq B_i(t)) & : \text{buy} \\ 0 & (B_i(t) > x_i(t) + \xi_i(t) > S_i(t)) & : \text{inactive} \\ -1 & (x_i(t) + \xi_i(t) \leq S_i(t)) & : \text{sell,} \end{cases} \quad (2)$$

where $B_i(t)$ and $S_i(t)$ represent threshold values to determine buying attitude and selling attitude at time t , respectively. $\xi_i(t)$ is the uncertainty of the i th dealer's decision-making. For simplicity it is assumed to be sampled from an identical and independent Gaussian distribution,

$$p(\xi_i) = \frac{1}{\sqrt{2\pi}\sigma_\xi} \exp\left(-\frac{\xi_i^2}{2\sigma_\xi^2}\right), \quad (3)$$

where $\sigma_\xi (> 0)$ is a standard deviation of $\xi_i(t)$. Of course this assumption can be weakened. Namely we can extend the uncertainty in the case of non-Gaussian noises and even correlated noises.

The excess demand is given by the sum of investment attitudes over the market participants,

$$r(t) = N^{-1} \sum_{i=1}^N y_i(t), \quad (4)$$

which can be an order parameter. Furthermore the market price $P(t)$ moves to the direction to the excess demand

$$\ln P(t + \Delta t) = \ln P(t) + \gamma r(t), \quad (5)$$

where γ represents a liquidity constant and Δt is a sampling period.

The dealers determine their investment attitude based on exogenous factors (fundamentals) and endogenous factors (market price changes). Generally speaking, the prediction of the i th dealer $z_i(t)$ is determined by a complicated strategy described as a function with respect to historical market prices, $F_i(s, P(t), P(t - \Delta t), \dots)$. Following the Takayasu's first order approximation [11] we assume that $z_i(t)$ is given by

$$z_i(t) = a_i(t)(\ln P(t) - \ln P(t - \Delta t)) = \gamma a_i(t)r(t - \Delta t), \quad (6)$$

where $a_i(t)$ is the i th dealer's response to the market price changes.

It is assumed that the dealers' response can be separated by common and individual factors,

$$a_i(t) = \zeta(t) + \eta_i(t), \quad (7)$$

where $\zeta(t)$ denotes the common factor, and $\eta_i(t)$ the individual factor. Generally these factors are time-dependent and seem to be complicated functions of both exogenous and endogenous variables.

For simplicity it is assumed that these factors vary rapidly in the limit manner. Then this model becomes well-defined in the stochastic manner. We assume that $\zeta(t)$ and $\eta_i(t)$ are sampled from the following identical and independent Gaussian distributions, respectively:

$$P_\zeta(\zeta) = \frac{1}{\sqrt{2\pi}\sigma_\zeta} \exp\left(-\frac{(\zeta - a)^2}{2\sigma_\zeta^2}\right), \quad (8)$$

$$P_\eta(\eta_i) = \frac{1}{\sqrt{2\pi}\sigma_\eta} \exp\left(-\frac{\eta_i^2}{2\sigma_\eta^2}\right), \quad (9)$$

where a represents a mean of $\zeta(t)$, $\sigma_\zeta (> 0)$ a standard deviation of $\zeta(t)$, and σ_η a standard deviation of $\eta (> 0)$.

Since we regard the market activity as the number of tick quotes per unit time it should be defined as the sum of dealers' actions:

$$q(t) = \frac{1}{N} \sum_{i=1}^N |y_i(t)|. \quad (10)$$

The market activity $q(t)$ may be regarded as an order parameter.

4 Numerical simulation

This agent model has nine model parameters. We fix $N = 100$, $B_i(t) = 0.01$, $S_i(t) = -0.01$, $\gamma = 0.1$, $\sigma_\eta = 0.01$, and $a = 0.0$ throughout all numerical simulations. It is assumed that an exogenous periodic information to the

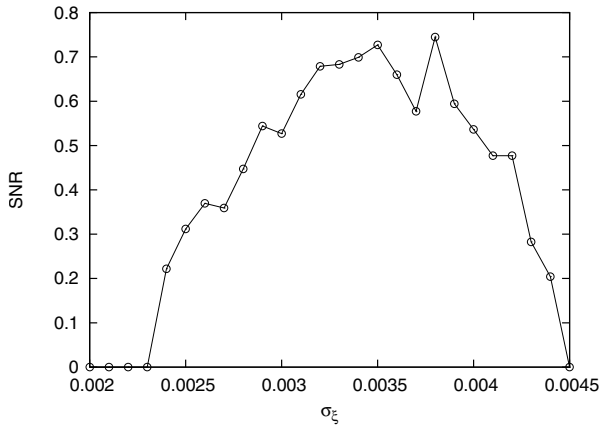


Fig. 4. Signal-to-noise ratio (SNR) obtained from power spectrum densities of $q(t)$ for the double-threshold agent model is plotted against the uncertainty of decision-making of agents at $N = 100$, $B_i = 0.01$, $S_i = -0.01$, $\gamma = 0.1$, $\sigma_\eta = 0.01$, $a = 0.0$, $\sigma_\zeta = 0.3$, $q_0 = 0.001$, $f = 0.8$, and $\Delta t = 1$.

market is subject to $s(t) = q_0 \sin(2\pi\Delta t f t)$ at $q_0 = 0.001$, $f = 0.8$ and $\Delta t = 1$.

We calculate the signal-to-noise ratio (SNR) of the market activity as a function of σ_ξ . The SNR is defined as

$$SNR = \log_{10} \frac{S}{N}, \quad (11)$$

where S represents a peak height of the power spectrum density, and N noise level.

From the numerical simulation we find non-monotonic dependency of the SNR of $q(t)$ on σ_ξ . Figure 4 shows a relation between the SNR and the noise strength σ_ξ . It has an extremal value around $\sigma_\xi = 0.0035$. Namely the uncertainty of decision-making plays a constructive role to enhance information transmission. If there are exogenous periodic information and the uncertainty of decision-making we can find the peak on power spectrum densities at appropriate uncertainty of decision-making due to stochastic resonance [12].

5 Conclusion

We analyzed time series of the number of tick quotes (market activity) and found there are short-time periodicities in the time series. The existence and positions of these peaks of the power spectrum densities depend on foreign currency markets and observation periods. The power spectrum densities have a peak at 2.5 min on the USD/JPY market, 3 min on the EUR/USD. There are some peaks at a few minutes on the JPY/EUR.

We developed the double-threshold agent model for financial markets where the agents choose three kinds of states and have feedback strategies to determine their decision affected by last price changes. From the numerical simulation we confirmed that the information transmission is enhanced due to stochastic resonance related to the uncertainty of decision-making of the market participants. We propose a hypothesis that the periodicities of the market activity can be observed due to stochastic resonance.

Appearance and disappearance of these peaks may be related to the efficiency of the markets. The efficiency market hypothesis [13] says that prices reflect information. Because quotes make prices tick frequency of the quotes can reflect information. If the peaks of the power spectrum densities come from exogenous information then SNR is related to the efficiency of the market. Namely the market may be efficient when the peaks appear.

The author thanks Prof. Dr. T. Munakata for stimulative discussions and useful comments. This work is partially supported by the Japan Society for the Promotion of Science, Grant-in-Aid for Scientific Research # 17760067.

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